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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

FREQUENT SAMPLING IN DISCRETE CHOICE

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Frequent Sampling in Discrete Choice

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Abstract

We analyse a discrete choice model based on random utility maximization involving frequent sampling, without restrictions on the stochastic structure (admitting dependence). For large samples, we calculate the (limiting) choice probabilities. They are hit for any size of the sample if and only if the invariance of achieved utility property holds. Examples are the Multinomial Logit model and the Generalised Extreme Value model.

KEYWORDS: Discrete choice, frequent sampling, random utility maximization, invariance of achieved utility, stochastic dependence.

1 Introduction

Many economic decisions comprise choice among discrete alternatives. Think of housing, workplace or the selection of a shopping center. To the observer or modeler, the decisions involve unobserved attributes of the alternatives and/or are subject to taste variations among the choice makers (McFadden (1981), Ben Akiva & Lerman (1985)). The random utility maximization model of discrete choice captures these features. A finite number of alternatives is indexed by $i \in \mathcal{A} = \{1, \ldots, m\}$ and the indirect utility of alternative i is given by a random variable, V_i. The joint-distribution F of $V = (V_1, \ldots, V_m)$ summarizes the frequencies of observed utilities and reflects the unobserved attributes and/or the taste variations. In an additive random utility model, indirect utility V_i has the additively separable form $V_i = U_i - c_i$ where $-c_i \in I\!R$ is the systematic part and U_i an error term. For example, alternatives may be destinations and c_i the travel cost to location i. Distribution F_0 assigned to the error vector $U = (U_1, \ldots, U_m)$ then determines the distribution F of V. We assume rational choice: the choice maker selects the alternative with the highest realized utility.

In a single sample framework, the choice maker observes a single realization of V and selects the "best" alternative. When the achieved utility is invariant in distribution

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across alternatives, the model has the invariance of achieved utility property (IAU). For an additive model Lindberg et al. (1990) provide a functional characterization of the distribution F_0 for the IAU property to hold. Examples are the *Multinomial Logit model* (MNL) and the *Generalized Extreme Value model* (GEV). The MNL model is the most widely used in empirical work, due to its computational simplicity. It has been derived from the axiom of Independence of Irrelevant Alternatives axiom (IIA) which states that the relative odds for any two alternatives are independent of the attributes or even the availability of any other alternative (Luce (1959)), but is subject to serious criticism (Debreu (1960)). In the MNL model, error terms U_i are stochastically independent and have type-1 extreme value (or Gumbel, or double exponential) distributions. The GEV model is more general and does not satisfy the IIA axiom (McFadden (1978)). The error terms have a multivariate extreme value joint distribution. It generates logit-like choice probabilities and allows patterns of dependence among the unobserved attributes of the alternatives. However, given the systematic parts $(-c_1, \ldots, -c_m)$, the GEV model is observationally equivalent to a model in which the error terms are independent and follow type-1 extreme value distributions, such as in the MNL model. (The parameters of the latter distributions depend on $(-c_1, \ldots, -c_m)$ and thus on the choice set; hence the IIA axiom is violated, see Jaïbi (1993).)

In a frequent sample framework, the choice maker samples several times before selecting an alternative. It is pertinent when each alternative is an aggregate of several opportunities, say through a grouping of locations, and when V_i is the indirect utility of one opportunity from alternative i, as in the model of product differentiation with perfectly free entry of Perloff and Salop (1985). In Jaïbi and ten Raa (1992a), the model of Perloff and Salop is generalized and it is proved that when utilities are independent across alternatives and the size of the sample goes to infinity, the choice probabilities are asymptotically Multinomial Logit (including the degenerate cases), irrespective the specification of the utilities' distributions.

This paper contains two results. We first provide a new characterization of the IAU property. It holds if and only if the choice probabilities are insensitive to the sample size. Moreover the IAU property is shown to be preserved under frequent sampling. MNL and GEV models are applications. Then we consider a frequent sampling model without any functional specification of the stochastic structure, admitting any dependence between the utilities. The limiting choice probabilities are calculated. This result extends the ones of Jaïbi and ten Raa (1992a-b).

The paper is organized as follows. Section 2 lays down the model. Section 3 provides the characterization of the IAU property with respect to frequent sampling. In Section 4, the general model with frequent sampling is analyzed.

2 Model description and notation

There are m discrete alternatives indexed by $i \in \mathcal{A} = \{1, \ldots, m\}$. Indirect utility of alternative i is V_i, a random variable. The utilities' vector $V = (V_1, \ldots, V_m)$ is assumed to have a continuous joint-distribution,

$$F(v_1, \ldots, v_m) = P\{V_1 < v_1, \ldots, V_m < v_m\} .$$

Associated with V are two random variables: *maximum utility* M and *best alternative* I, defined by

$$\begin{aligned} M &= \max_j V_j , \\ I &= i \;\; if \quad M = V_i . \end{aligned}$$

The probability of ties is assumed to be zero so that I is well-defined up to a negligible set. A sample is a realization of V. In a single sample framework, alternative i is selected with probability

$$p_{i,1} = P\{M = V_i\} = P\{I = i\}.$$

In a frequent sample framework, we denote the k-th sample by $V_{(k)} = (V_{1,k}, \ldots, V_{m,k})$ and $V_{i,k}$ is the (indirect) utility of alternative i for sample k. We assume the $V_{(k)}$'s to be independent random vectors having F as common distribution. With $V_{(k)}$ we associate the maximum utility $M_{(k)}$ and the best alternative $I_{(k)}$, defined like M and I, respectively.

The utility reached by alternative i after n samples is

$$\hat{V}_{i,n} = \max_{1 \leq k \leq n} V_{i,k}$$

and the maximum utility over all alternatives is

$$\hat{M}_n = \max_{1 \leq i \leq m} \hat{V}_{i,n} .$$

We denote by $\hat{I}_n$ the best alternative out of the n samples. Thus $\hat{I}_n = i$ if $\hat{M}_n = \hat{V}_{i,n}$, and alternative i is chosen with probability

$$p_{i,n} = P\{\hat{M}_n = \hat{V}_{i,n}\} = P\{\hat{I}_n = i\} .$$

When the sample size goes to infinity, the limiting choice probabilities, if they exist, are denoted by

$$p_i = \lim_{n \to \infty} p_{i,n}, \quad i \in \mathcal{A} .$$

3 The Invariance of Achieved Utility

Before we consider the IAU property in general, let us review two models.

In the *Multinomial logit model* (MNL), error terms U_i are assumed to be independent and to follow type-1 extreme value distributions

$$P\{U_i < u\} = \exp(-A_i \, e^{-\mu u}) .$$

Here $A_i \geq 0$ is a parameter specific to alternative i and $\mu > 0$ is common to all the alternatives. When systematic parts $(-c_1, \ldots, -c_m)$ are added to the error terms, the choice probabilities are

$$p_{i,1} = \frac{A_i\, e^{-\mu\, c_i}}{\sum_{j=1}^{m} A_j\, e^{-\mu\, c_j}}, \quad i \in \mathcal{A}. \tag{1}$$

Conversely, (1) holds if and only if the U_i's are independent and are type-1 extreme value distributed (cf. Yellot (1977)).

In the *Generalized extreme value model* (GEV), error vector U follows a multivariate extreme value distribution with p.d.f.

$$F_0(u_1, \ldots, u_m) = \exp(-G(e^{-\mu\, u_1}, \ldots, e^{-\mu\, u_m})).$$

Here $\mu > 0$ is a parameter and G is a non-negative, linearly homogeneous function with continuous mixed partial derivatives (non-positive even and non-negative odd mixed partial derivatives) such that $\lim_{x_j \to \infty} G(x_1, \ldots, x_m) = \infty$ for all j. When systematic parts $(-c_1, \ldots, -c_m)$ are added to the error terms, the choice probabilities are

$$p_{i,1} = \frac{e^{-\mu\, c_i} G_i(e^{-\mu\, c_1}, \ldots, e^{-\mu\, c_m})}{G(e^{-\mu\, c_1}, \ldots, e^{-\mu\, c_m})}, \quad i \in \mathcal{A}, \tag{2}$$

where G_i is the i-th partial derivative of G. The GEV model reduces to the MNL model when $G(x_1, \ldots, x_m) = \sum_{j=1}^{m} A_j x_j$. It also reduces to the Nested Multinomial Logit model when

$$G(x_1, \ldots, x_m) = \sum_{l=1}^{k} \left(\sum_{j \in \mathcal{A}_l} A_j x_j^{\theta_l^{-1}} \right)^{\theta_l}$$

where $(\mathcal{A}_l)_{l=1,\ldots,k}$ is a partition of $\mathcal{A}$ and where each parameter $\theta_l \neq 1$ introduces a correlation among the alternatives in $\mathcal{A}_l$ (McFadden (1978)). The GEV model accommodates patterns of dependence between unobserved attributes of the alternatives. For a discussion of this point we refer to Jaïbi (1993).

The MNL and GEV models have the IAU property, which we shall discuss now. For each alternative i such that $P\{I = i\} > 0$ the distribution of the achieved utility is given by the conditional distribution $P\{M < u \mid I = i\}$. The IAU property is said to hold if this conditional distribution does not depend on the alternative, that is if

$$\forall u \in \mathbb{R} \quad , \quad P\{M < u \mid I = i\} = P\{M < u\}.$$

This condition is equivalent to the stochastic independence between the maximal utility M and the best alternative I:

$$\forall i \in \mathcal{A}, \;\; \forall u \in \mathbb{R}, \;\; P\{I = i\,,\; M < u\} = P\{I = i\}\; P\{M < u\}\,. \tag{3}$$

For additive models, the IAU property holds if and only if the distribution of the error terms, F_0, has the following functional form

$$F_0(u_1, \ldots, u_m) = \varphi(G(e^{-\mu\, u_1}, \ldots, e^{-\mu\, u_m}))$$

where $\mu > 0$ is a parameter, G any non-negative linearly homogeneous function on $\mathbb{R}_+^m$ and φ any function on $\mathbb{R}_+$ such that F_0 is indeed a p.d.f. (Lindberg et al. (1990)). This functional form generalizes the GEV one and generates choice probabilities given by the same formula (2).

In the following, we provide a characterization of the IAU property in terms of frequent sampling. No particular structure is imposed on the utilities. Any random utility model is said to be *insensitive to frequent sampling* if the choice probabilities are constant in the sample size:

$$\forall\, i \in \mathcal{A},\ \ \forall\, n > 1 \qquad p_{i,n} = p_{i,1}\,.$$

Proposition 1 *The IAU property holds in a random utility model if and only if the model is insensitive to frequent sampling.*

Proof. Because the probability of ties is zero and by a symmetry argument on independent and identically distributed $V_{(k)}$, for any $n > 0$

$$\begin{aligned} P\{\hat{I}_n = i\,,\ \hat{M}_n < v\} &= P\left(\cup_{l=1}^{n}\{I_{(l)} = i\,,\ \hat{M}_n = M_{(l)} < v\}\right) \\ &= nP\left\{I_{(1)} = i\,,\ M_{(k)} < M_{(1)} < v \ for\ 1 < k \le n\right\} \\ &= n\int_{-\infty}^{v} \Pi_{k=2}^{n} P\{M_{(k)} < u\}\, dP\{I_{(1)} = i\,,\ M_{(1)} < u\} \\ &= n\int_{-\infty}^{v} P\{M < u\}^{n-1}\, dP\{I = i\,,\ M < u\}\,. \end{aligned} \tag{4}$$

Suppose that the IAU property holds. By (3)

$$\begin{aligned} P\{\hat{I}_n = i\,,\ \hat{M}_n < v\} &= P\{I = i\}\, n\int_{-\infty}^{v} P\{M < u\}^{n-1}\, dP\{M < u\} \\ &= p_{i,1} P\{M < v\}^{n} \\ &= p_{i,1} P\{\hat{M}_n < v\}\,. \end{aligned} \tag{5}$$

It implies that

$$\begin{aligned} p_{n,i} &= \lim_{v\to\infty} P\{\hat{I}_n = i\,,\ \hat{M}_n < v\} \\ &= p_{i,1} \lim_{v\to\infty} P\{\hat{M}_n < v\} \\ &= p_{i,1}\,. \end{aligned}$$

Thus the model is insensitive to the frequent sampling.

Conversely, for each i with $P\{I = i\} > 0$, taking limits in (4) provides

$$\begin{aligned} p_{i,n} &= n\int_{-\infty}^{\infty} P\{M < u\}^{n-1}\, dP\{I = i\,,\ M < u\} \\ &= p_{i,1}\, n\int P\{M < u\}^{n-1}\, dP\{M < u \mid I = i\} \end{aligned}$$

When the model is insensitive to frequent sampling, $p_{i,n} = p_{i,1}$ and, therefore,

$$n\int P\{M < u\}^{n-1}\, dP\{M < u \mid I = i\} = 1$$

for all $n \geq 1$. This implies that for all $i \in \mathcal{A}$ with $P\{I = i\} > 0$ and for all $u \in \mathbb{R}$

$$P\{M < u \mid I = i\} = P\{M < u\} .$$

as shown in the Lemma in Appendix 1. Thus the IAU property holds. □

As a corollary, we obtain that the IAU property is preserved by the aggregation of opportunities.

Corollary 2 *If the IAU property holds for a single sample framework, it also holds for a multi sample framework.*

Proof. By Proposition 1 $p_{i,n} = p_{i,1}$. Thus for each v (5) reads

$$\begin{aligned} P\{\hat{I}_n = i \, , \, \hat{M}_n < v\} &= p_{i,n} P\{\hat{M}_n < v\} \\ &= P\{\hat{I}_n = i\} \, P\{\hat{M}_n < v\} . \end{aligned}$$

This is the IAU property for sample size n. □

4 A general model with frequent sampling

In this section we consider a general random utility model with frequent sampling, without any functional specification on the utilities' distribution. Thus, any pattern of dependence across alternatives is admitted. Our purpose is to calculate the limiting choice probabilities. We make the following technical assumption:
p.d.f. F is continuous and is such that the limits

$$q_i = \lim_{u \to b} \frac{P\{M \geq u \; , \; I = i\}}{P\{M \geq u\}} , \tag{6}$$

are well defined, $i \in \mathcal{A}$, where $b = \sup\{u : \; P\{M < u\} < 1\} \leq \infty$.
This assumption is similar to the comparability of tails of distributions introduced in Jaïbi and ten Raa (1992b). The IAU property ensures it, as Example 1 below will show, but the assumption is far more general and merely rules out pathological distributions. Note that by the continuity of F the probability of ties is zero, and for all n it holds

$$\sum_{i=1}^{m} p_{i,n} = 1 \; , \quad \textit{and} \quad \sum_{i=1}^{m} q_i = 1 .$$

Our main result is the following.

Theorem 3 *Under the above assumption, the limiting choice probabilities exist and coincide with the limits q_i defined by (6):*

$$p_i = q_i \; , \quad i \in \mathcal{A}.$$

Proof. The proof is an adaptation of the proof of the theorem in Jaïbi and ten Raa (1992b) and deferred to the Appendix.

The limiting choice probabilities are easily related to the single sample choice probabilities. Let β_i be *the thickness of the upper tail of the distribution of the achieved utility for alternative i relative to that of the maximal utility,* defined for each i with $P\{I = i\} > 0$ by

$$\begin{aligned} \beta_i &= \lim_{u \to b} \frac{P\{M \geq u \mid I = i\}}{P\{M \geq u\}} \\ &= \lim_{u \to b} \frac{P\{V_i \geq u \mid I = i\}}{P\{M \geq u\}}. \end{aligned}$$

It is straightforward that $q_i = P\{I = i\}\, \beta_i$. Thus we have

$$p_i = p_{i,1}\, \beta_i\,.$$

In comparison to the single sample choice, frequent sampling favours the alternatives with an achieved utility distribution having a thick upper tail, that is with a large β_i. For large samples, the (limiting) choice probabilities are multiplied by the relative thicknesses β_i.

Examples.
1. When the IAU property holds (as for MNL and GEV models), the choice probabilities are constant in the sample size and thus $p_i = p_{i,1}$. On the other hand, for each i the limit q_i is well defined. Indeed, by the IAU property the term under the limit in (6) is constant in u and equals $p_{i,1} = P\{I = i\}$. Thus Theorem 3 follows. Alternatively, for each i with $p_{i,1} > 0$, the relative thickness β_i is equal to one.
2. When the utilities V_i to be independent with respective distributions F_i, the p.d.f. F^* of $M = \max_i V_i$ is the product $\Pi_{i=1}^m F_i(u)$. Then, if the F_i's have comparable upper tails in the sense that the limits

$$\alpha_i = \lim_{u \nearrow b} \frac{1 - F_i(u)}{1 - F^*(u)}$$

are well defined ($b = \sup\{v :\ F^*(v) < 1\}$), the limiting choice probabilities are given by these limits, i.e. $p_i = \alpha_i$, $i \in \mathcal{A}$ (Jaïbi and ten Raa (1992b)). This is coherent with our result because $q_i = \alpha_i$ ($i = 1, \ldots, m$) in this case (see Appendix 3). If moreover F^* has a regular upper tail in the sense that

$$\lim_{u \to \infty} \frac{1 - F^*(u + v)}{1 - F^*(u)}$$

exists for each $v > 0$, the limiting choice probabilities have a Multinomial Logit representation (including the degenerate cases): there exists a parameter μ revealed by F^* and $(c_1, \ldots, c_m)$ revealed by the laws F_i such that

$$p_i = \frac{\exp(-\mu c_i)}{\sum_{j=1}^m \exp(-\mu c_j)}$$

(Jaïbi and ten Raa (1992b)). These are the limiting choice probabilities of the additive model with systematic utilities $-c_i$ and with independent error tems having $F^{\bullet}$ as common distribution (Jaïbi and ten Raa (1992a)).

5 Conclusion

The invariance of achieved utility property holds in a random utility maximization model if and only if the choice probabilities are insensitive to the sample size. The property is preserved under frequent sampling. MNL and GEV models are applications. When sampling affects the choice probabilities, the latter tend to a limit as the samples become large. The limiting choice probabilities are equal to the single sample choice probabilities multiplied by the corresponding thickness of the upper tail of the achieved utility distribution.

Appendix

1. Lemma *Two continuous p.d.f. α and β such that $n \int \alpha^{n-1}\, d\beta = 1$ for all $n \geq 1$ are identical.*

Proof. Let Y be a random variable having β as p.d.f.. For each $n \geq 0$

$$\begin{aligned} E\left(\alpha^n(Y)\right) &= \int \alpha^n(x)\, d\beta(x) \\ &= \frac{1}{n+1} \end{aligned}$$

This implies that the random variable $\alpha(Y)$ follows the uniform distribution on $[0,1]$. Indeed, it has as Laplace transform

$$\begin{aligned} E\left(e^{-s\alpha(Y)}\right) &= \int \sum_{n\geq 0} (-1)^n \frac{s^n}{n!} \alpha^n(y)\, d\beta(y) \\ &= \sum_{n\geq 0} (-1)^n \frac{s^n}{n!} \int \alpha^n(y)\, d\beta(y) \\ &= \sum_{n\geq 0} (-1)^n \frac{s^n}{n!} \frac{1}{n+1} \\ &= \frac{1-e^{-s}}{s}. \end{aligned}$$

Hence, for any $y \in I\!R$, $\beta(y) = P\{Y < y\} = P\{\alpha(Y) < \alpha(y)\} = \alpha(y)$. □

2. Proof of Theorem 3. For ease of notation, let $\tilde{M}(u) = P\{M > u\}$ and $\tilde{M}_i(u) = P\{I = i\,,\ M > u\}$. Fix $i \in \mathcal{A}$, such that

$$q_i = \lim_{u \to b} \frac{\tilde{M}_i(u)}{\tilde{M}(u)} > 0\,,$$

with $b = \sup\{u : \tilde{M}(u) > 0\}$. Because $q_i > 0$, $b_i = \sup\{u : \tilde{M}_i(u) > 0\} = b$. The continuity of $\tilde{M}$ implies that of $\tilde{M}_i$. From equation (??) we have

$$\begin{aligned} p_{i,n+1} &= (n+1)\int \left(1-\tilde{M}\right)^n(u)\, d\left(P\{I=i\}-\tilde{M}_i(u)\right) \\ &= (n+1)\int \left(1-\tilde{M}\right)^n(u)\, d\left(-\tilde{M}_i(u)\right) \\ &= (n+1)\int_{-\infty}^{b} \exp\left(n\log(1-\tilde{M}(u))\right) d(-\tilde{M}_i(u)) \end{aligned}$$

For each $\epsilon > 0$, there exist b^i_ϵ with $\tilde{M}_i(b^i_\epsilon) > 0$ such that for $b^i_\epsilon < u < b$,

$$\tilde{M}(u) \le (q_i^{-1}+\epsilon)(\tilde{M}_i(u))\,.$$

It follows that

$$\begin{aligned} p_{i,n+1} &\ge (n+1)\int_{b^i_\epsilon}^{b} \exp\left(n\log(1-\tilde{M}(u))\right) d(-\tilde{M}_i(u)) \\ &\ge (n+1)\int_{b^i_\epsilon}^{b} \exp\left(-nC_\epsilon(q_i^{-1}+\epsilon)(1-\tilde{M}_i(u))\right) d(-\tilde{M}_i(u)) \\ &= \frac{n+1}{n}\frac{1}{C_\epsilon(q_i^{-1}+\epsilon)}\left(1-\exp\left(-nC_\epsilon(q_i^{-1}+\epsilon)\tilde{M}_i(b^i_\epsilon)\right)\right) \end{aligned}$$

where $C_\epsilon = -\epsilon^{-1}\log(1-\epsilon)$, and where the last inequality is obtained by the concavity of $\log(1-x)$ on $[0,\epsilon]$. Hence, for any $\epsilon > 0$,

$$\liminf_{n\to\infty} p_{i,n} \ge \frac{1}{C_\epsilon(q_i^{-1}+\epsilon)}$$

Thus

$$\begin{aligned} \liminf_{n\to\infty} p_{i,n} &\ge \liminf_{\epsilon\downarrow 0} \frac{1}{C_\epsilon(q_i^{-1}+\epsilon)} \\ &= q_i \end{aligned}$$

because $\lim_{\epsilon\downarrow 0} C_\epsilon = 1$. The fact that the q_i's add up to unity implies that the limiting choice probabilities p_i exist and equal q_i, respectively. □

3. Proof of $q_i = \alpha_i$, Example 2. When the V_i's are independent with respective distributions F_i,

$$\begin{aligned} P\{M > u\,,\, I=i\} &= \int_u^\infty \Pi_{k\neq i}F_k(v)dF_i(v) \\ &= (1-F_i(u)) - \int_u^\infty (1-\Pi_{k\neq i}F_k(v))\,dF_i(v)\,. \end{aligned}$$

Dividing both sides by $P\{M > u\}$, we obtain

$$\frac{P\{M > u\,,\, I=i\}}{P\{M>u\}} = \frac{1-F_i(u)}{1-F^*(u)} - \int_u^\infty \frac{1-\Pi_{k\neq i}F_k(v)}{1-F^*(u)}dF_i(v)\,. \tag{7}$$

But the integral in (7) tends to 0 as u goes to ∞ because for all $v \geq u$,

$$\begin{aligned} \frac{1 - \Pi_{k \neq i} F_k(v)}{1 - F^*(u)} &\leq \frac{1 - \Pi_{k \neq i} F_k(u)}{1 - F^*(u)} \\ &\leq 1 \end{aligned}$$

Thus $q_i = \alpha_i$ by taking limits in (7). Moreover, in the independent case, comparability of the tails of the distributions F_i and our technical assumption are equivalent. □

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